SOME REGULARITIES FOR A COOLED CURRENT LEAD

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An analytical study is presented for leads cooled by the vapor from the boiling coolant. Graphs are presented for leads with a temperature difference between room temperature and helium temperature. It is found that the model is of sufficient accuracy for design calculations.

Low-temperature cryostats often use current leads cooled by vapor, and this considerably reduces the boiling loss. These cooled leads have been the subject of several studies [1-10], but there is no theory that describes all the observed properties, and such a theory should describe reasonably accurately a suitable means of designing them. The present study attempts to fill this gap.

1. Cooled leads are used mainly for superconducting devices, so one can avoid heat release in part of the lead directly adjoining the liquid coolant by using superconductors [1]. The resistive part of the lead above the superconducting one should constitute a good heat exchanger in contact with the vapor. One can represent the specific resistance of the metal as a function of temperature via a linear relationship to a first approximation; the thermal conductivity is assumed constant, and then the assumption of ideal heat transfer* gives the differential equation for the steady-state heat-flux distribution in the lead as

$$\frac{d^2T}{dx^2} - M\frac{dT}{dx} + \left(\frac{I}{2}\right)^2 T = 0. \tag{1}$$

The quantity β appearing in the equation can be derived from

$$\int_{T_1}^{T_2} \rho(T)\,dT = \beta \int_{T_1}^{T_2} T\,dT. \tag{2}$$

We solve (1) to find the temperature distribution in the lead with the boundary conditions $T = T_1$ at $x = 0$ and $T = T_2$ at $x = 1$, the result being

$$\frac{T(x)}{T_2} = \frac{\sin\left(\frac{x}{2}\sqrt{I^2-M^2}\right)}{\sin\left(\frac{1}{2}\sqrt{I^2-M^2}\right)} \exp\left(M\frac{x-1}{2}\right) + \frac{T_1}{T_2}\,\frac{\sin\left(\frac{1-x}{2}\sqrt{I^2-M^2}\right)}{\sin\left(\frac{1}{2}\sqrt{I^2-M^2}\right)} \exp\left(M\frac{x}{2}\right). \tag{3}$$

Here $I > M$; if $I < M$, the sines are replaced by hyperbolic functions.† In practice, $T_2 \gg T_1$, so the lead temperature is determined mainly by the first term in (3). Here we use (3) without the second term, which is justified for $\rho(T_1) \ll \rho(T_2)$ and is equivalent to neglecting the Joule heat at the cold and relative to the heat produced in the rest of the lead.

The thermal-conduction equation gives the heat flux into the liquid as

$$Q = \lambda s dT(x)/dx\,|_{x=0}. \tag{4}$$

We substitute for T(x) from (3) to get a formula for Q in the general case where M and I may take any values; on the other hand, these quantities are linked in a cryostat with such leads, since the evaporation rate is determined by Q:

*The heat-exchanger design is not considered here.

†This applies to all the trigonometric functions appearing below.

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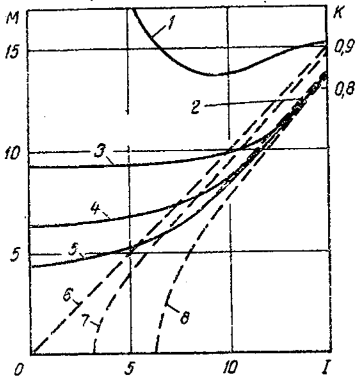

Fig. 1

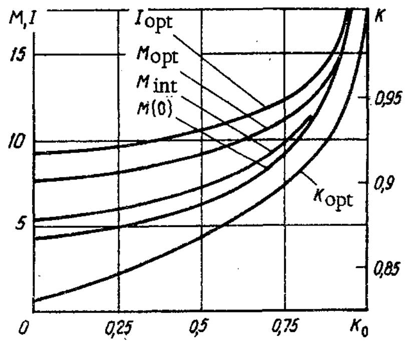

Fig. 2

Fig. 1. Dynamic characteristics: 1) $K(K_0 = 0)$; 2) $M_{opt} = f(I_{opt})$; 3) $M(K_0 = 0.75)$; 4) $M(K_0 = 0.5)$; 5) $M(K_0 = 0)$; 6) $M = 1$; 7) $M_{cr\ max}$; 8) $M_{cr\ min}$.

Fig. 2. Characteristic points of the dynamic characteristic in relation to K_0.

$$\eta (m - m_0) = Q. \tag{5}$$

We equate (4) and (5) to get the transcendental equation

$$K - K_0 = \frac{D \sqrt{1 - K^2} \exp(-M/2)}{2 \sin (M \sqrt{K^{-2} - 1}/2)} . \tag{6}$$

If D is given, this equation defines the functions $K = f(I)$ and $M = f(I)$ (the latter is called the dynamic characteristics of the lead). We get a family of these functions for various m_0.

We see from (6) that $K \to 1$ for $I \to \infty$, i.e., the dynamic characteristics tend asymptotically to the linear function $M = I$ (Fig. 1). This agrees with relationships obtained by experiment [2] and from theory [3] on the assumption that $\rho = \text{const}$, $\lambda = \text{const}$. From the definition of K for $i \gg i_{opt}$ we have $m \simeq 2i\sqrt{\beta\lambda}/c$; if the properties of the metal follow the Wiedemann–Franz law then $\sqrt{\beta\lambda} = \pi k/e\sqrt{3} = \text{const}$, and the evaporation rate is dependent only on the current and the specific heat of the vapor.

The evaporation rate falls when the current is switched off, but it remains higher than m_0 and is determined by the thermal conductivity of the leads.

If the current is greater than the optimal value, there is a temperature rise (T_{max}/T_2) in the leads defined by (3), in which

$$x = \frac{\pi - \operatorname{arctg}(\sqrt{I^2 - M^2}/M)}{\sqrt{I^2 - M^2}/2} \tag{7}$$

is the coordinate of the point in the lead having the highest temperature. In these formulas, M and I must be taken in accordance with the appropriate dynamic characteristic.

By opposing various conditions on (6) we have obtained a series of relationships defining the behavior of the characteristic points as K_0 varies; the resulting equations were solved graphically. It was assumed that $\Delta T = T_2 - T_1 \sim T_2 = 300°K$, $\eta/c = 3.95°K$, which corresponds to the current input conditions for leads working between room temperature and liquid helium. Figures 1-5 give the results.

Figure 1 shows the following functions: three dynamic characteristics for $K_0 = 0$; 0.5; 0.75; the asymptote $M = I$, to which these tend for $I \to \infty$; a graph for the shift in the optimal point $M_{opt}(I_{opt})$ as K_0 increases; and two graphs for the critical values of M and I (see below on critical values). The dynamic characteristics show how m increases if i rises from 0 to ∞ for various m_0; here K decreases from ∞ to its least value K_{opt}, on the way passing through unity and then rising asymptotically towards unity.

Figure 2 shows how the characteristic points vary with K_0; the functions $M_{opt}(K_0)$ and $I_{opt}(K_0)$ give the displacement of the optimal point at which the straight line from the origin of slope $K_{opt}(K_0)$ touches the dynamic characteristic. The function $M_{int}(K_0)$ defines the behavior of the point of intersection between the dynamic characteristic and the asymptote $M = I$, while $M(0) = f(K_0)$ does the same for the intersection with the ordinate axis.

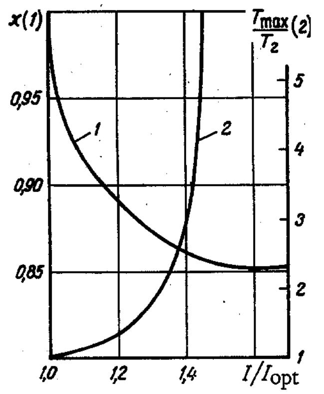

Fig. 3

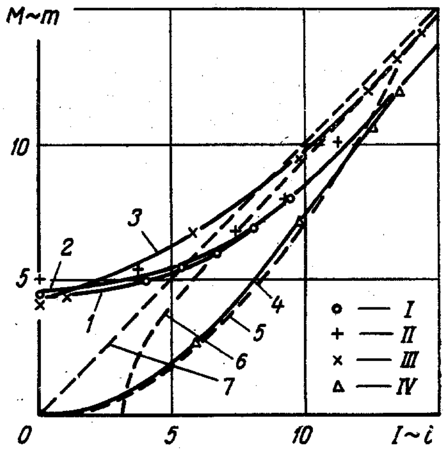

Fig. 4

Fig. 3. Coordinate of the hottest point and temperature rise for $K_0 = 0$.

Fig. 4. Comparison of calculations with published data in dimensionless form: 1 and 2) lines representing calculations for K_0 of 0 and 0.1 respectively, points from [1]; 1, I) $i_{opt} = 2 \times 1320$ A; 2, II) $i_{opt} = 2 \times 500$ A; 3) points from [2], line calculated for $m_0 = f(i)$ via the law of curve 4; 3, III) $i_{opt} = 2 \times 10$ A; 4) points for M_0 calculated from the experimental points 3 via Fig. 1, $M_0 = f(I)$; 5) quadratic relationship closest to the law of curve 4, $M_0 = 0.07 I^2$; 6) $M_{cr\,max} = f(I)$; 7) $M = I$.

If $I > I_{opt}$, the dynamic characteristic (Fig. 1) very rapidly approaches the $M_{cr.\,min}(I)$ value, while remaining above it, but being below $M_{opt}(I_{opt})$; if the above properties are known and we have available the functions shown in Fig. 2, we can construct the dynamic characteristic for any K_0 without solving (6).

Figure 3 shows (3) and (7) as calculated for $K_0 = 0$, with the value of x in (3) taken from (7). It is clear that currents in excess of the optimal cause the distance from the cold end to the hottest point to diminish, the result being 0.5 of the lead length when $i/i_{opt} \approx 1.6$. If the current is increased further, the peak temperature tends asymptotically towards the hot end. The excess temperature rise (T_{max}/T_2) increases fairly rapidly, and this factor becomes about 3 for $i/i_{opt} = 1.41$ and about 6 for $i/i_{opt} = 1.45$.

2. In some cases, the highest current has to be supplied only periodically (for instance, when a frozen-in field is used with a superconducting solenoid). In that case, the overall loss of coolant can be reduced by optimizing the current lead to some smaller current. This problem can be handled as follows if we assume that c, λ, and β are known:

1) By specifying the following quantities: m_0, i_{max}, T_{max};

2) Taking T_{max}/T_2 from a curve analogous to Fig. 3, we determine $I/I_{opt} = i/i_{opt}$ and hence i_{opt}, the current for which the lead should be optimized;

3) We calculate K_0, and from Fig. 2 we determine K_{opt}, I_{opt}, M_{opt}, $M(0)$;

4) We calculate $(l/s)_{opt}$, m_{opt}, $m(0)$;

5) Knowing $I/I_{opt} = i_{max}/i_{opt}$ and I_{opt}, we calculate the I corresponding to i_{max}. We use the graph for the dynamic characteristic corresponding to this K_0 to determine M and calculate the evaporation rate m at i_{max}.

3. The experimental evidence and the formulas of [3] give us for fairly high currents that $dm/di \simeq \sqrt{\rho\lambda/c\eta} = 0.648 \cdot 10^{-4}$ g/sec·A, where we have taken $\rho\lambda = 0.45 \cdot 10^{-6}$ W·Ω/°K to bring about agreement with experiment, this corresponding to the mean temperature of the lead of about 30°K. In that case, the temperature in the upper part of the lead considerably exceeds room temperature, which restricts the current.

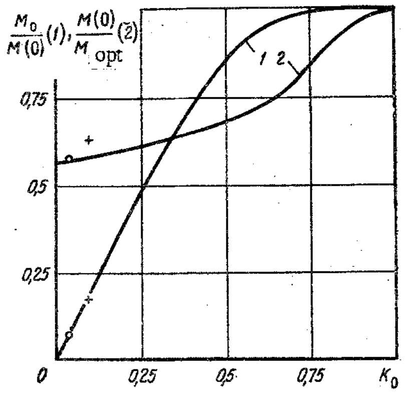

Fig. 5. Calculated values for some ratios and observed values of [1].

Figure 1 shows that currents larger than the optimal value produce slopes approximately equal to unity, so $dm/di \simeq 2\sqrt{\beta\lambda}/c = 2\pi k/ce\sqrt{3} = 0.602 \cdot 10^{-4}$ g/sec · A.

The ratio $(m/i)_{opt} = 2.71 \cdot 10^{-3}$ l/h · A has been obtained [1] as the same for two sets of leads made of electrotechnical copper working at optimal currents $i_{opt} = 2 \cdot 500$ A and $i_{opt} = 2 \cdot 1320$ A; the factor 2 is introduced because a set of leads consists of two such. The value $m_0 = 0.15$ liter/h = const, was determined by the cryostat used. These values were used to calculate $\beta\lambda = 2.17 \cdot 10^{-8}$ W · Ω/°K²; $K_0 = 0.035$; $\lambda s/l = 0.0865$ W/°K for the 2 × 1320 A set and $\beta\lambda = 2.14 \times 10^{-8}$ W/Ω/°K²; $K_0 = 0.0927$; $\lambda s/l = 0.0314$ W/°K for the 2 × 500 A one. Then the observed values were converted to the dimensionless M and I, which may be compared with the calculated values in Fig. 4. The observed $m(0)/m_{opt}$ and $m_0/m(0)$ for these two sets of leads are shown by points on the curves for the calculated $M(0)/M_{opt} = f(K_0)$ and $M_0/M(0) = f(K_0)$ (Fig. 5).

The observed m(i) of [2] are shown in Fig. 4 in dimensionless form. In the conversion we took all the dimensional coefficients in M and I as in [2]. Two points may be noted: the observations do not coincide with the curves calculated on the assumption that m_0 = constant, and also there was no damage to the leads on using a current more than 1.5 times the calculated optimum value (paper components were used in this design of lead). Both features are due to the resistive part of the lead directly in contact with the liquid coolant, which was below the heat exchanger (cooled lead proper) and did not exchange heat efficiently with the vapor. The resulting Joule heat in this part went to evaporation. In that case, $m_0 = f(i)$, and then increases in I caused M to deviate from any single dynamic characteristic (Fig. 1) and to pass from a curve of lower K_0 to one of higher K_0. If the point with coordinates M and I in Fig. 1 does not fall below the function $M_{opt}(I_{opt})$, then the leads do not become excessively heated, as Fig. 4 shows. The M(I) for various K_0 of Figs. 1 and 2 enable one to calculate $M_0(I)$, which explains these observations. One can combine Figs. 1 and 4 to determine values for K_0 and I for each point. The values $M_0 = K_0 I$ are shown in Fig. 4, together with the approximating relationship $M_0 = 0.07\, I^2$ (subject to the condition $\eta m_0 = Ri^2$, where R = constant). Then $R = 16 M_0 \eta \beta l/scI^2 \approx 0.25 \cdot 10^{-3}$ Ω.

Then R acts as an automatic source of cold vapor to cool the leads and tends to suppress the excessive heating which enables one to attain $m/m(0) = 3.45$; this property can be utilized for the purposes formulated in the preceding section.

These comparisons show that $\beta\lambda$ can be taken as $2.15 \cdot 10^{-8}$ W · Ω/°K² for electrotechnical copper, but it is not clear from experiment what values should be selected for β and λ. If we use the listed data of [11] and assume $\rho_{4.2°K} = 0.03$ μΩ · cm, we get from (2) that $\beta = 0.512 \cdot 10^{-8}$ Ω · cm/°K, and the corresponding value is $\lambda = 4.2$ W/cm · °K.

4. A characteristic feature of the calculation is that one uses only the single numerical ratio $D = \Delta Tc/\eta$; the calculated functions differ little for different kinds of materials used for such leads, and the relationship is essentially governed by the linear temperature dependence of the product of the specific resistance and the thermal conductivity.

The increment in the evaporation rate with the current is approximately constant at currents exceeding the optimal value; it is dependent in the main on the specific heat of the vapor, and is largely independent of the lead material and is quite independent of the natural boiling rate of the cryostat, and also of the length and cross section of the leads, as well as of the temperature at the hot end of the lead and the latent heat of evaporation.

The condition for an optimum in the lead $dQ/d(l/s) = 0$ is equivalent to the specification $dT/dl|_{x=l} = 0$, so there is no heat influx from the hot end of the lead in the optimum case, and the liquid coolant receives the power difference

$$Q = P_L - mc\Delta T. \tag{8}$$

If $i > i_{opt}$ the temperature gradient at the hot end increases very rapidly, as does the heat flux along the lead.

Figure 5 also shows that m(0) and m_{opt} become indistinguishable from m_0 as K_0 increases; physically this means that the current is so small and the specific heat of the vapor is so large that (8) tends to zero. Then $(l/s)_{opt} \to \infty$ (M and $I \to \infty$), and this result is obtained on account of neglect of the second term in (3). However, one may assume that the combined leads suggested in [3] will correspond precisely to our model for zero resistance of the cold end. Then the above arguments apply, and for $K_0 \geq 1$ one can design a lead that produces practically no additional boiling of the coolant in the cryostat.

We get a dual inequality from the trigonometric functions in the above formulas:

$$\pi < \sqrt{I^2 - M^2} < 2\pi$$

if I and M are such that the square root equals 2π, then Q and T_{max} become indefinitely large, as (3) and (4) show; the function $M_{cr\,min}(I)$ has been derived from this condition. When the root equals π, Q does not attain its optimum value, and the corresponding function is $M_{cr\,max}(I)$; Fig. 1 shows that $K_0 = \text{const}$ and $I > I_{opt}$ give the interval between $M_{opt}(I_{opt})$ and $M_{cr\,min}(I)$ as fairly small, while that between M(I) and $M_{cr.min}(I)$ is even less. The practical conclusion is that any accidental reduction or uneven distribution of the vapor passing through the lead causes a marked increase or marked nonuniformity in the heating.

The point of intersection of $M_{cr.max}(I)$ with the abscissa I ($I = \pi$, Fig. 1) defines $(l/s)_{opt}$ for an uncooled lead (here doubling the current should inevitably burn out such a lead, since then $I = 2\pi$). The evaporation rate for such a lead in the optimum case is $m_{di\,opt} = m_0 + i\Delta T\sqrt{\beta\lambda}/\eta$. As we have $m_{coo\,opt} = K_{opt}2i\sqrt{\beta\lambda}/c$ for a cooled lead in the optimum case, the ratio of these rates is $m_{di\,opt}/m_{coo\,opt} = (K_0 + D/2)/K_{opt}$ and is about 49.5 for $K_0 = 0$ and about 39 for $K_0 = 1$.

We have presented above mainly the methods of analysis and calculation for such leads. Lack of space has forced us to omit formulas that confirm the conclusions but the following comments may usefully be made:

Exact solution of the topic in section 2 requires a series of curves as in Fig. 3 for various K_0, while the sequence of calculations should contain a series of operations 3) and 2) for each new K_0;

One should distinguish $K_0 = M_0/I$ in (6) and $K_0 = M_0/I_{opt}$ in the functions shown in the figures;

The K_{opt} of [1] for leads handling 2×500 and 2×1320 A cannot be taken as general results because the K_0 for these were different which may represent an experimental error.

NOTATION

$T(x)$	is the lead temperature as a function of dimensionless coordinate;
T_1, T_2, T_{max}	are the temperatures of the cold and hot ends of the lead and maximum temperature (exceeding T_2) at some point on the lead;
x	is the proportion of the lead length reckoned from the cold end;
l	is the lead length;
s	is the cross section;
i	is the current;
$\rho(T)$	is the real temperature dependence of the specific resistance;
β	is the factor in the linear approximation for the temperature dependence of the specific resistance;
λ	is the thermal conductivity;
η	is the latent heat of evaporation;
c	is the specific heat of vapor;
Q	is the heat flux from the lead into the coolant;
m	is the coolant evaporation rate;
m_0	is the coolant evaporation rate determined by all heat sources other than Q (this includes the part of the lead not cooled by the vapors if this has a resistance and is in contact with the liquid);
$m(0)$	is the evaporation rate in a cryostat at zero current;
R	is the resistance of uncooled part of lead;
$\rho_{4.2^\circ K}$	is the residual resistance;
P_J	is the Joule heat produced throughout the lead;
k	is the Boltzmann's constant;
e	is the electronic charge; and $\pi = 3.14$.

Dimensionless Quantities

$D = c\Delta T/\eta$	is the ratio of specific heats;
$M = (c/\lambda)(l/s)m$	is the evaporation-rate parameter;
$I = 2\sqrt{\beta/\lambda}\,l/si$	is the current parameter;
$K = M/I = (c/2\sqrt{\beta\lambda})(m/i)$	is the ratio of evaporation rate to current,

$M_0 = (c/\lambda)(l/s)m_0$;

$M(0) = (c/\lambda)(l/s)m(0)$;

$M_{opt} = (c/\lambda)[(l/s)m]_{opt}$;

$I_{opt} = 2\sqrt{(\beta/\lambda)}\,[(l/s)i]_{opt}$;

$K_{opt} = (c/2\sqrt{\beta\lambda})(m/i)_{opt}$;

$K_0 = (c/2\sqrt{\beta\lambda})(m_0/i)$.

LITERATURE CITED

1. Éfferson, Pribory dlya Nauch. Issled., No. 12, 82 (1967).
2. J. E. C. Williams, Cryogenics, 3, 234 (1963).
3. V. E. Keilin and E. Yu. Klimenko, Cryogenics, 6, 222 (1966).
4. E. Ya. Kazovskii and V. P. Kartsev, Inzh.-Fiz. Zh., 9, No. 1, 96 (1965).
5. Sobol and McNichol, Pribory dlya Nauch. Issled., No. 4, 61 (1962).
6. S. Deiness, Cryogenics, 5, 269 (1965).
7. P. W. Matthews, T. L. Khoo, and P. D. Neufeld, Cryogenics, 5, 213 (1965).
8. L. Donadieu and C. Dammann, Revue General de l'Electricite, 77, No. 2, 193 (1968).
9. A. B. Pippard, Cryogenics, 5, 81 (1965).
10. E. Carbonell, M. Renord, and P. Berard, Cryogenics, 8, 314 (1968).
11. M. P. Malkov, I. B. Danilov, A. G. Zel'dovich, and A. B. Fradkov, Handbook on the Technical Physics of Cryogenics [in Russian], Gosénergoizdat, Moscow–Leningrad (1963).